A hybrid metaheuristic for production planning

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Introduction

This work deals with two problems arising in production planning:

- **lot sizing**
- **scheduling**

- usually these problems are treated separately
- for both problems: exact solution can be rather hard
- appropriate solvers are different:
  - lot sizing $\longrightarrow$ mixed integer programming (MIP)
  - scheduling $\longrightarrow$ constraint programming
- metaheuristics: provide a unified framework
- this work: focus on the *integration*

Motivation

- Practical problem:
  - large industry
  - stable demand
  - production site where raw materials are transformed into end products.
- Currently:
  - scheduling operations come from customer orders
  - scheduling based on feasibility: no notion of cost involved
  - demand is stable $\longrightarrow$ why not think about lot sizes?
- Aim:
  - formalise the problem
  - lot sizing + scheduling $\longrightarrow$ scheduling operations derived from good/optimal lot sizes
  - implement a prototype
  - check feasibility of the approach with nearly-real data
- Planning:
  - Short term (scheduling): monthly basis
  - Medium term (lot sizing): yearly basis

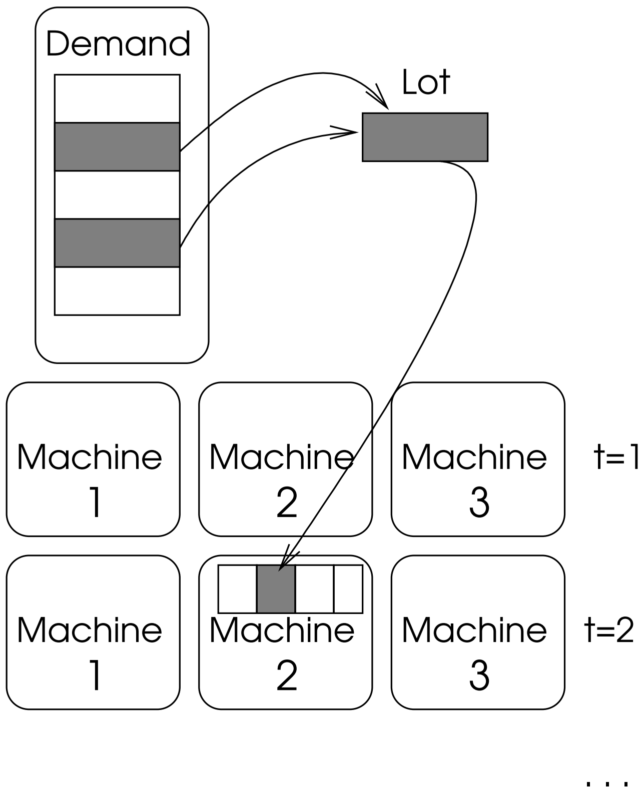
Background

Previous work in this area: *LISCOS* European project

- Exact approaches
- MIP for lot sizing
- Constraint programming for scheduling
- Both are commercial solvers
- Cost $\longrightarrow$ not appropriate for prototyping

$\longrightarrow$ metaheuristics

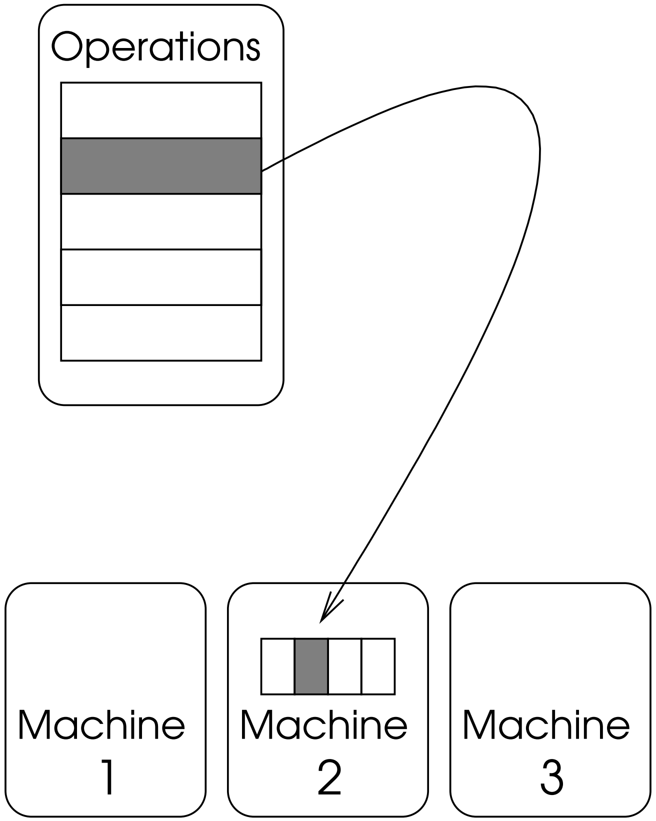
Lot sizing



Considering all the orders, for the whole of the planning horizon, decide:

- quantity of each lot to be produced
- when to produce each lot
- (not concerned with *order of production* in the machines)

Scheduling

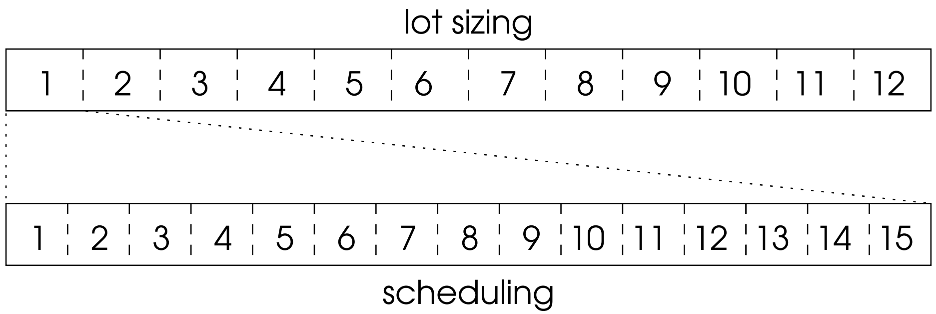


For each operation of a given period of the lot sizing problem:

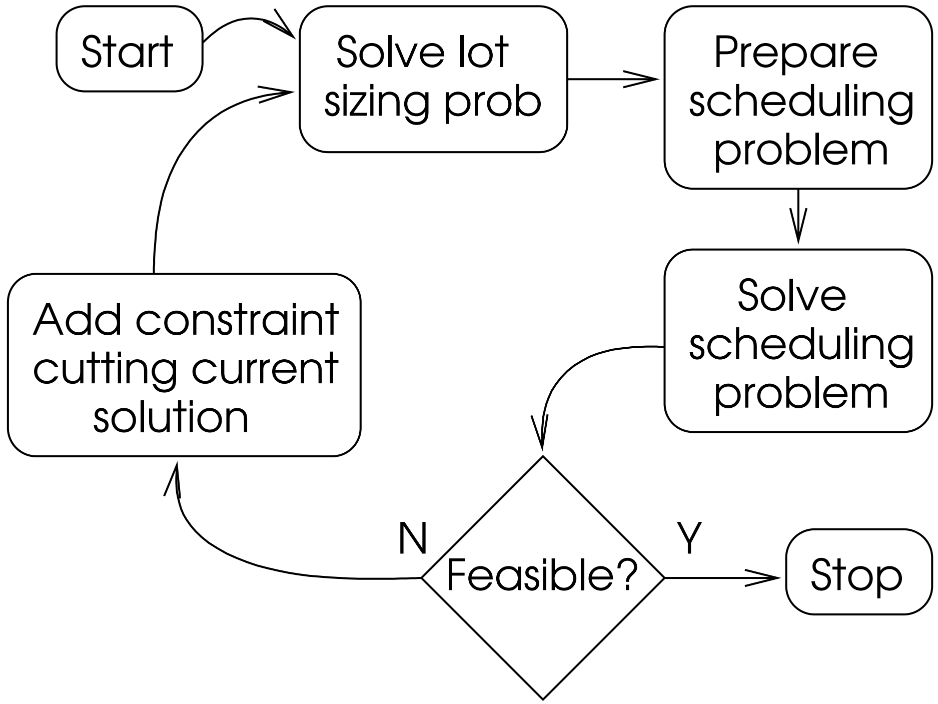
- assign it to a machine
- assign it an order in the operations of that machine
- detail: machines can operate in several *modes*:
  - full capacity ⟶ higher cost
  - reduced capacity ⟶ lower cost

Time horizons

- are different for lot sizing and for scheduling
- horizon for scheduling ↔ one period of lot sizing model
- usually: scheduling only for the first period of lot sizing



Main solution procedure



Lot sizing model

- Costs:
  - setup (fixed) costs
  - variable production costs
  - inventory
  - backlog
- Decision varibles:
  - manufacture or not of a product in each period: setup, binary variable y_{pmt}
    - $y_{pmt} = 1$ if product p is manufactured in machine m during period t
    - $y_{pmt} = 0$ otherwise
  - amount produced: continuous variable x_{pmt}
    - corresponding to y_{pmt}.
    - $x_{pmt} > 0 \Rightarrow y_{pmt} = 1$
  - inventory h_{pt} and backlog g_{pt}

Objective

setup costs: $F = \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} f_{pmt}\ y_{pmt}$

- f_{pmt} is the cost of setting up machine m on period t for producing p

variable costs: $V = \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} v_{pmt}\ x_{pmt}$

- v_{pmt} is the variable cost of production of p on machine m, period t

inventory costs: $I = \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} i_{pt}\ h_{pt}$

- h_{pt} is the amount of product p that is kept in inventory at the end of period t
- i_{pt} is the unit inventory cost for product p on period t

backlog costs: $B = \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} b_{pt}\ g_{pt}$

- g_{pt} is the amount of product p that failed to meet demand at the end of period t
- b_{pt} is the unit backlog cost for product p on period t.

objective: $\text{minimise}\, z = F + V + I + B$

Constraints:

flow conservation:

$$h_{p,t-1} - g_{p,t-1} + \sum_{m \in \mathcal{M}^p} x_{pmt} = D_{pt} + h_{pt} - g_{pt} \qquad \forall\ p \in \mathcal{P},\ \forall\ t \in \mathcal{T}.$$

h_{p0}, h_{pT}: initial and final inventory

g_{p0}, g_{pT}: initial and final backlog

time availability on each period:

$$\sum_{p \in \mathcal{P} : m \in \mathcal{M}^p} \left(\frac{x_{pmt}}{\gamma_{pm}} + \tau_{pmt}\ y_{pmt} \right) \leq A_{mt} \qquad \forall\ m \in \mathcal{M},\ \forall\ t \in \mathcal{T}.$$

γ_{pm} is the total capacity of production of product p on machine m per time unit

τ_{pmt} is the setup time required if there is production of p on machine m during period t

A_{mt} is the number of time units available for production on machine m during period t.

setup constraints:

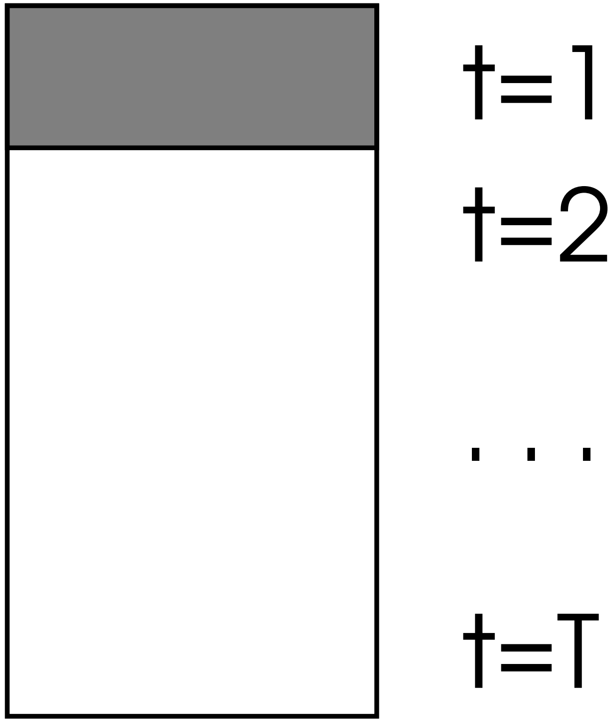
$$x_{pmt} \leq \gamma_{pm}\ A_{mt}\ y_{pmt}$$

$$
\begin{aligned}
\text{minimise} \quad & z = F + V + I + B \\
\text{subject to :} \quad & F = \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} f_{pmt}\ y_{pmt} \\
& V = \sum_{p\in\mathcal{P}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} v_{pmt}\ x_{pmt} \\
& I = \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} i_{pt}\ h_{pt} \\
& B = \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} b_{pt}\ g_{pt} \\
& h_{p,t-1} - g_{p,t-1} + \sum_{m\in\mathcal{M}^p} x_{pmt} = D_{pt} + h_{pt} - g_{pt}, \quad \forall\ p \in \mathcal{P},\ \forall\ t \in \mathcal{T} \\
& \sum_{p\in\mathcal{P}: m\in\mathcal{M}^p} \left(\frac{x_{pmt}}{\gamma_{pm}} + \tau_{pmt}\ y_{pmt} \right) \leq A_{mt}, \quad \forall\ m \in \mathcal{M},\ \forall\ t \in \mathcal{T} \\
& x_{pmt} \leq \gamma_{pm}\ A_{mt}\ y_{pmt} \quad \forall\ p \in \mathcal{P},\ \forall\ m \in \mathcal{M}^p,\ \forall\ t \in \mathcal{T} \\
& F, V, I, B \in \mathbb{R}^+ \\
& h_{pt},\ g_{pt} \in \mathbb{R}^+, \quad \forall\ p \in \mathcal{P},\ \forall\ t \in \mathcal{T} \\
& x_{pmt} \in \mathbb{R}^+,\ y_{pmt} \in \{0, 1\}, \quad \forall\ p \in \mathcal{P}, \forall\ m \in \mathcal{M},\ \forall\ t \in \mathcal{T}
\end{aligned}
$$

Construction: relax-and-fix-one-product

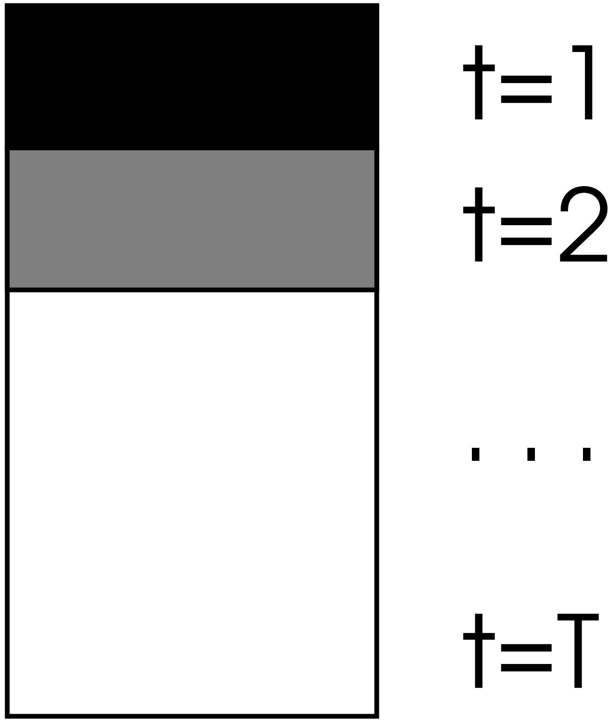
- **construction of a solution:** based on partial relaxations of the initial problem
- variant of the classic relax-and-fix heuristic

Relax-and-fix



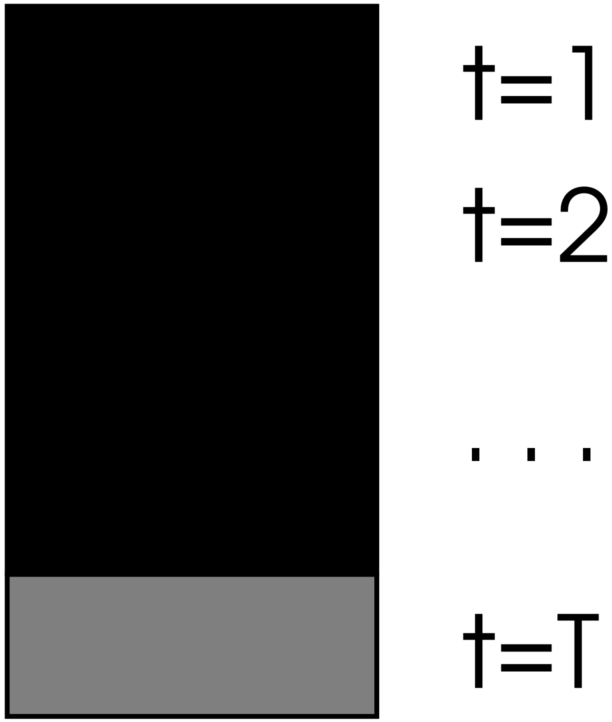
- each period is treated independently
- relax all the variables except those of period 1:
  - keep y_{pm1} integer
  - relax integrity for all other y_{pmt}
- solve this MIP, determining heuristic values for $\bar{y}_{pm1}$

Relax-and-fix



- each period is treated independently
- relax all the variables except those of period 1:
  - keep y_{pm1} integer
  - relax integrity for all other y_{pmt}
- solve this MIP, determining heuristic values for $\bar{y}_{pm1}$
- move to the second period:
  - variables of the first period are fixed at $y_{pm1} = \bar{y}_{pm1}$
  - variables y_{pm2} are integer
  - and all the other y_{pmt} relaxed
- this determines the heuristic value for y_{pm2}

Relax-and-fix



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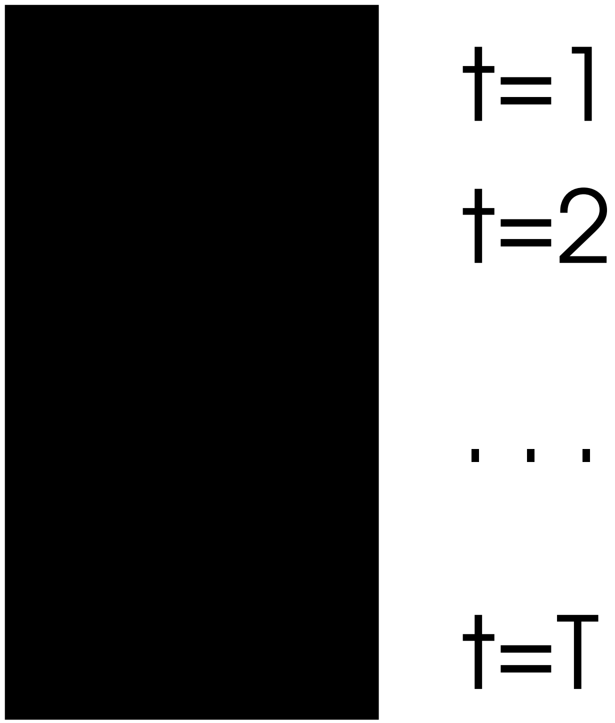
- these steps are repeated, until all the y variables are fixed

Relax-and-fix



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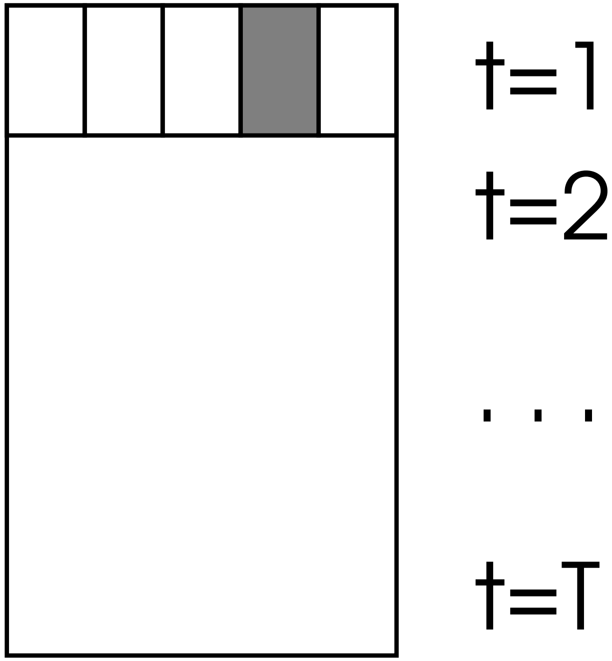
Relax-and-fix heuristic.

- reported to provide very good solutions for many lot sizing problems
- however, for large instances the exact MIP solution of even a single period can be too time consuming
- we propose a variant were each MIP determines only the variables of one period *that concern a single product* $\longrightarrow$ **relax-and-fix-one-product**

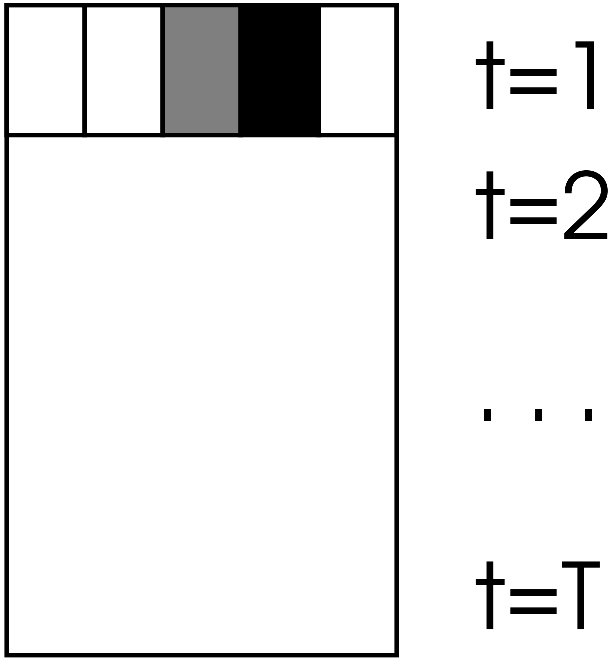
Relax-and-fix-one-product variant.



RelaxAndFixOneProduct()

relax all y_{pmt} as continuous variables
for $t = 1$ **to** T
  foreach $p \in \mathcal{P}$
    foreach $m \in \mathcal{M}^p$
      set y_{pmt} as integer
    solve MIP$\longrightarrow$ $\bar{y}_{pmt}, \forall m \in \mathcal{M}^p$
    foreach $m \in \mathcal{M}^p$
      fix $y_{pmt} := \bar{y}_{pmt}$
return $\bar{y}$

Relax-and-fix-one-product variant.



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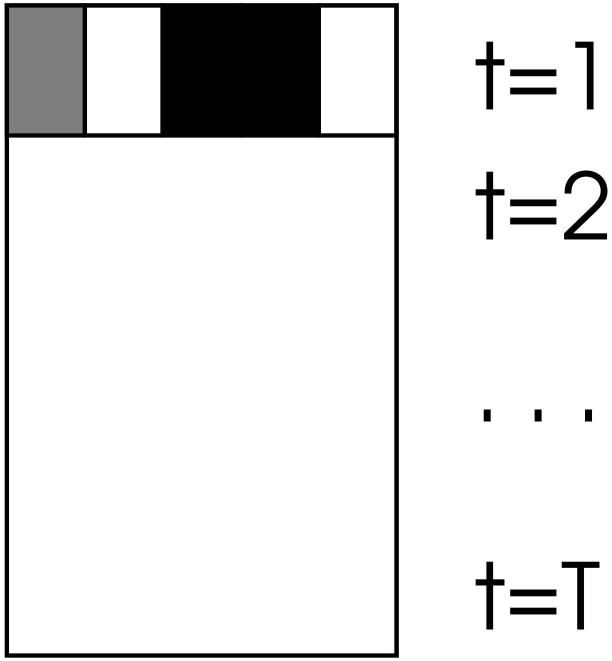

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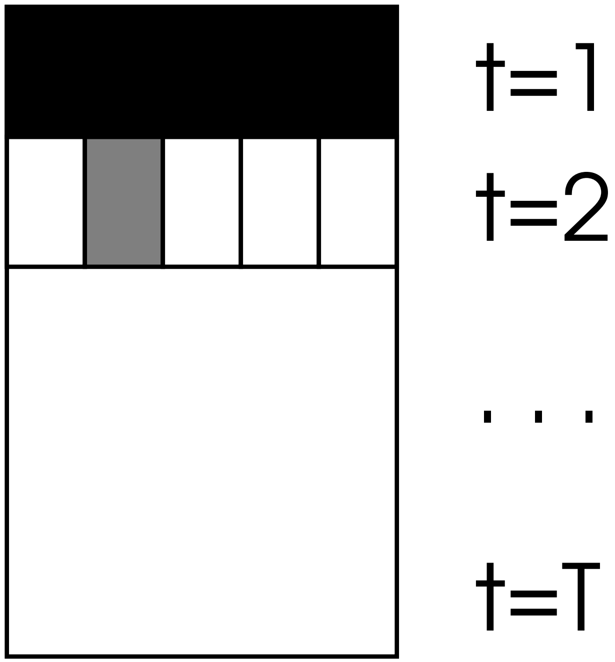

RELAXANDFIXONEPRODUCT()

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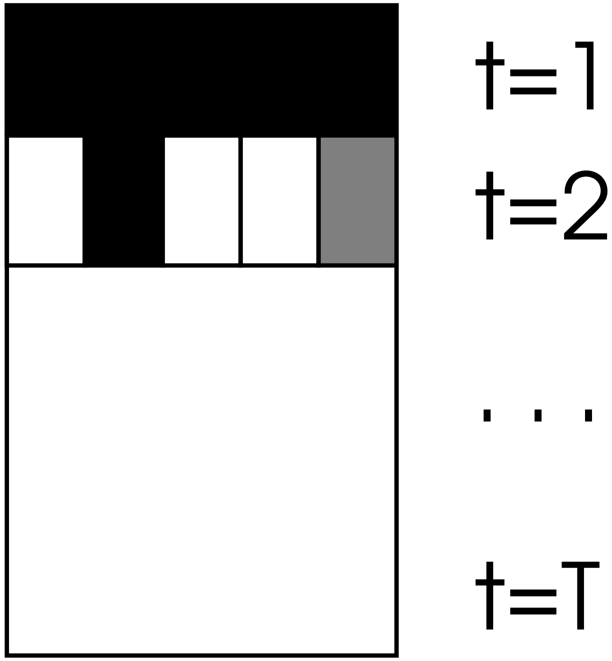

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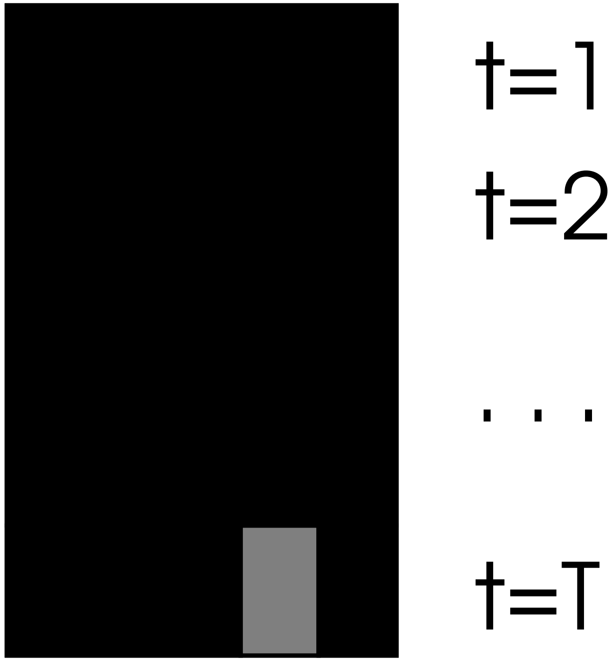

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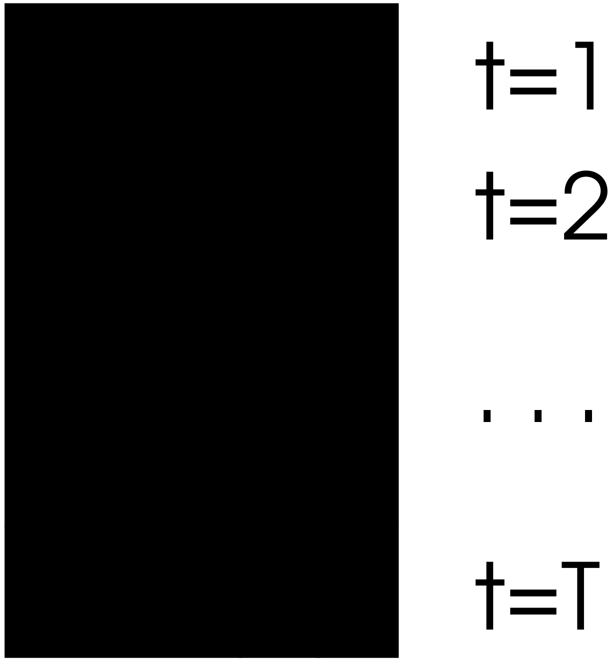

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return $\bar{y}$

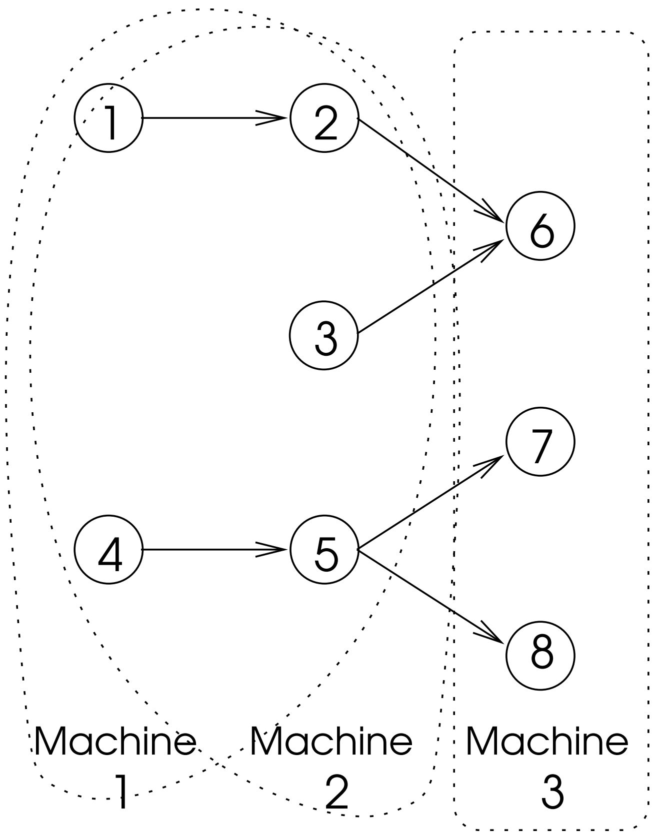
Additional advantage: if repeated, can produce different solutions
$\longrightarrow$ repeat it a number of times, retain the best found solution

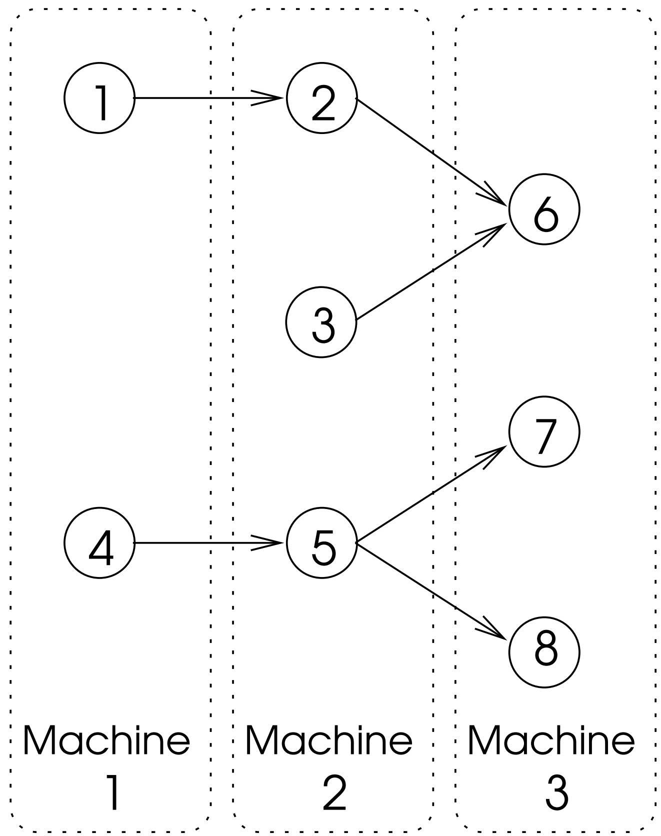
Scheduling: solution representation

There are two decisions that have to be taken for specifying a scheduling solution:

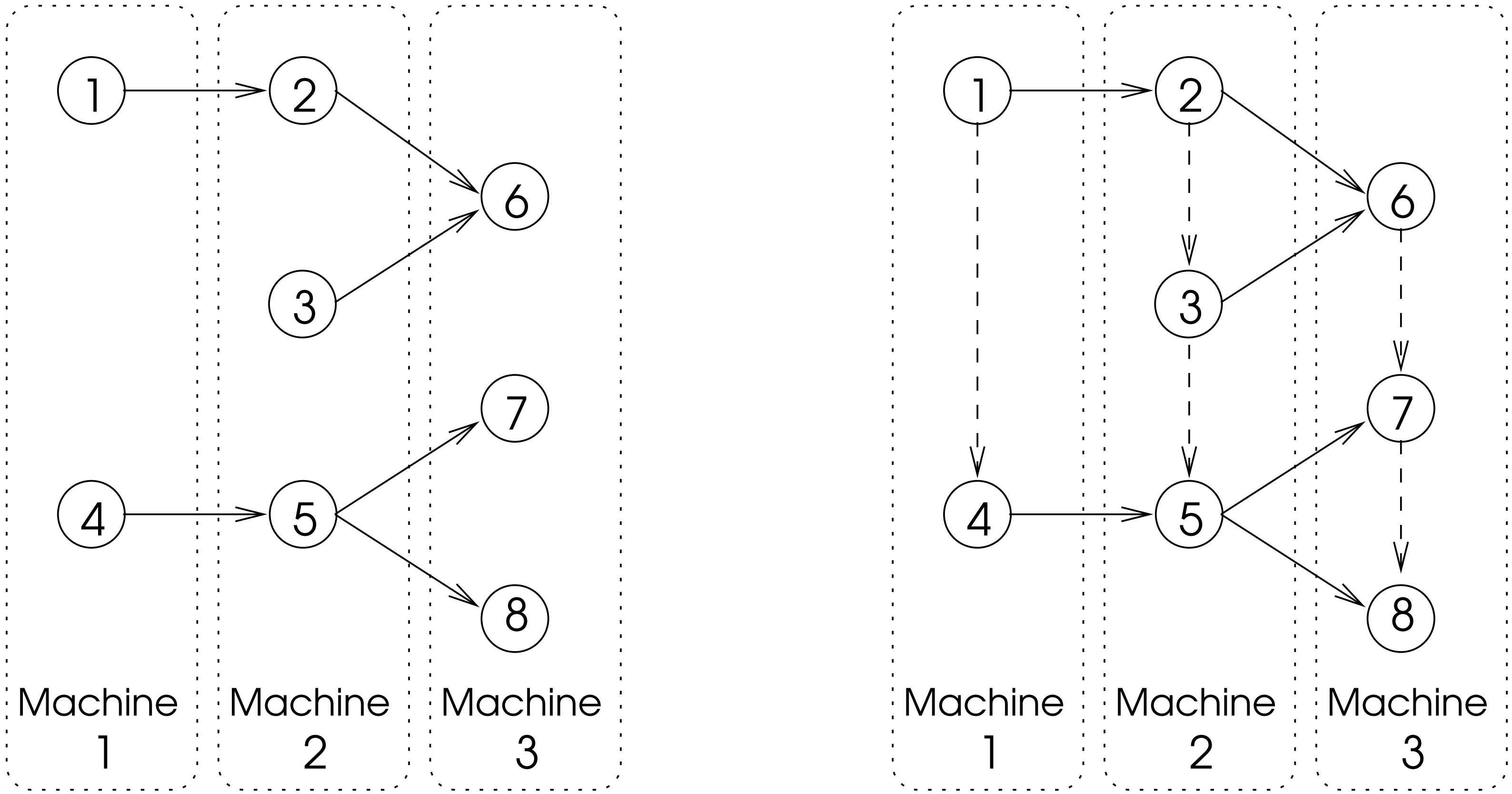
- Assigning a machine to each operation
- Establish an order for the operations inside each machine

Assigning a machine to each operation

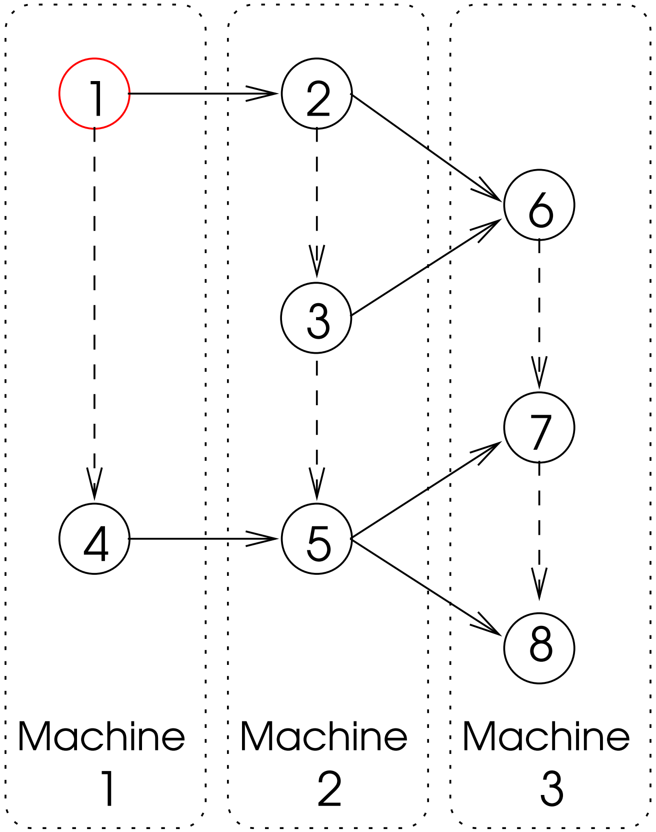




Operation order for each machine

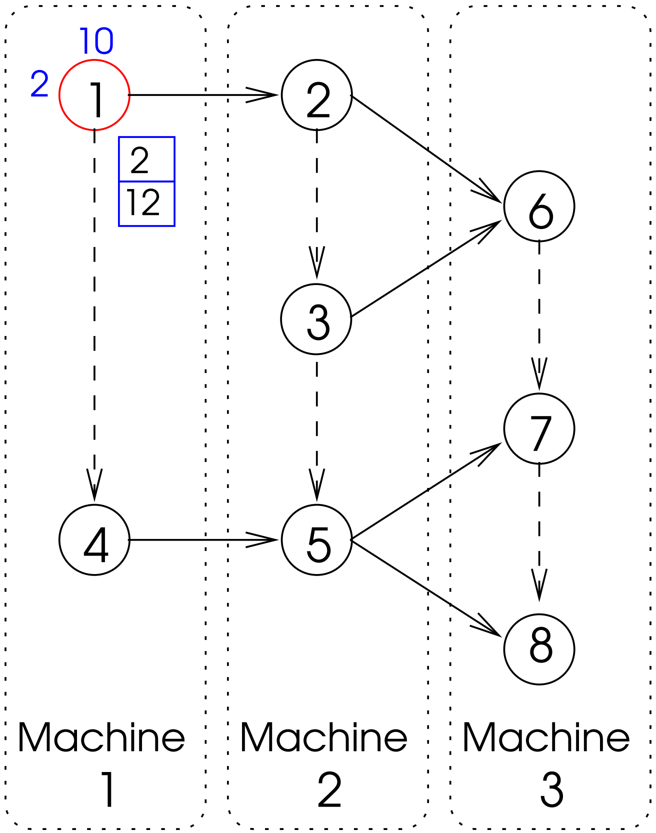


Solution evaluation (computing makespan and cost)



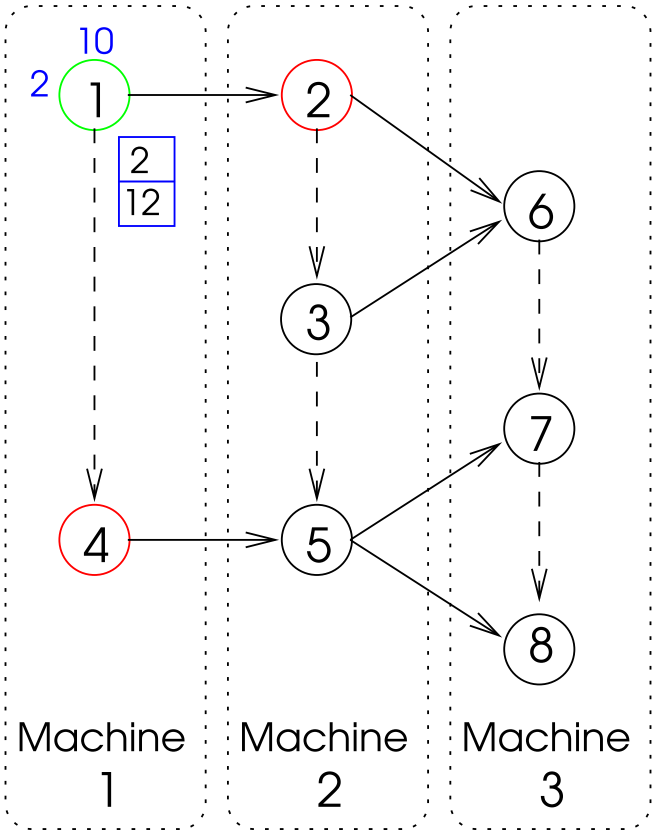
- Start scheduling operations which do not have free (unscheduled) predecessors

Solution evaluation



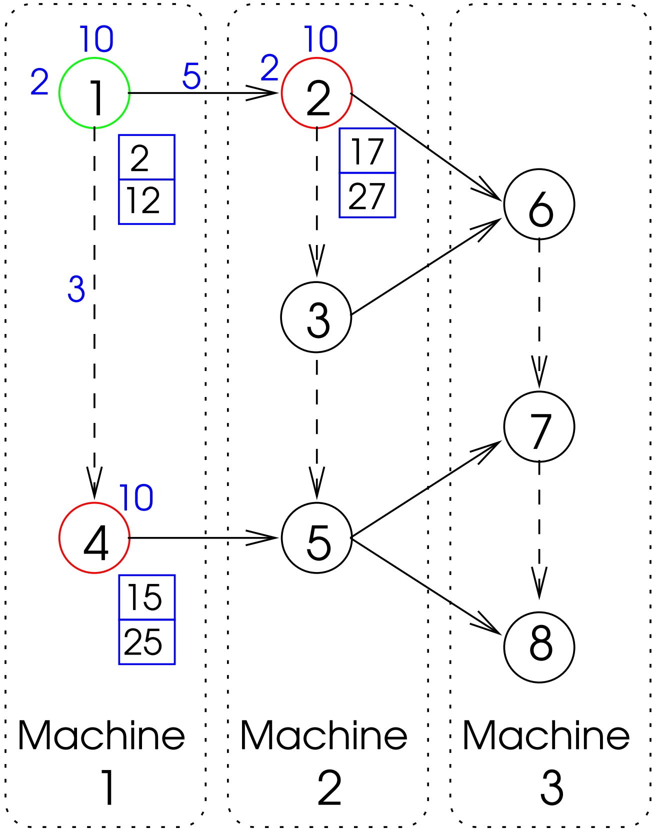
- Start scheduling operations which do not have free (unscheduled) predecessors
- Fix their earliest start time and earliest finish time

Solution evaluation



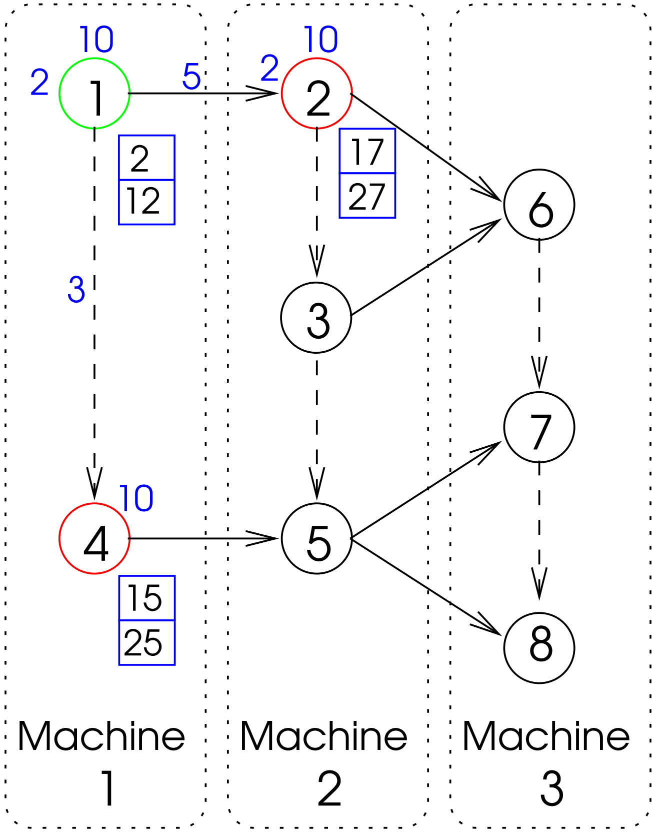
- Start scheduling operations which do not have free (unscheduled) predecessors
- Fix their earliest start time and earliest finish time
- Check operations which can now be scheduled

Solution evaluation



- Start scheduling operations which do not have free (unscheduled) predecessors
- Fix their earliest start time and earliest finish time
- Check operations which can now be scheduled
- Fix their start and finish times
- . . .

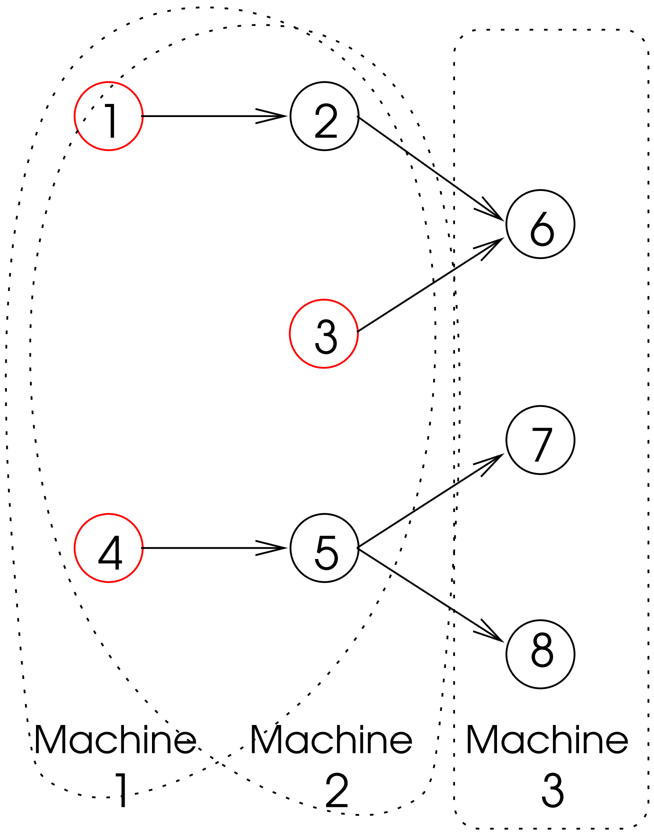
Solution evaluation



- Start scheduling operations which do not have free (unscheduled) predecessors
- Fix their earliest start time and earliest finish time
- Check operations which can now be scheduled
- Fix their start and finish times
- . . .

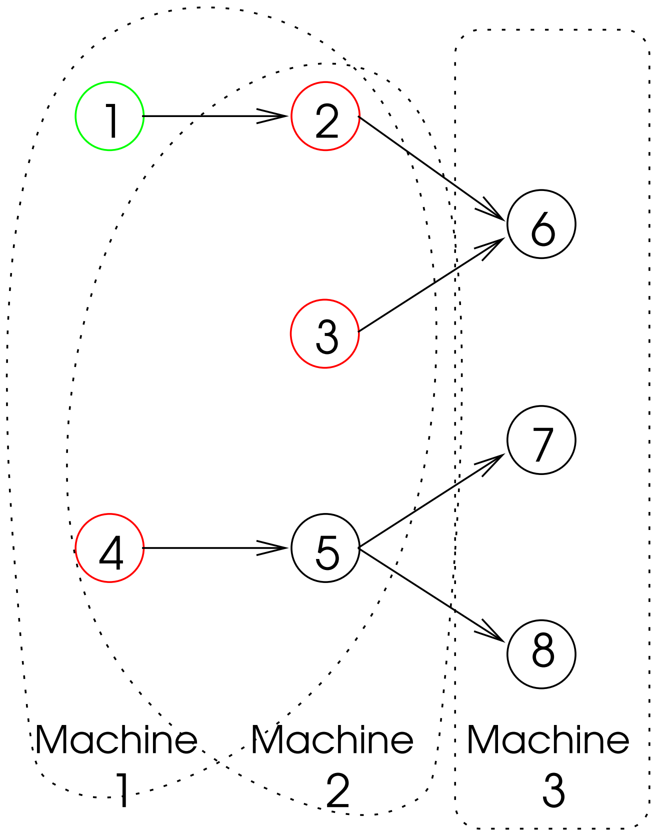
- **changeover** times/costs
- **transfer** times/costs
- **fixed/variable productions** times/costs

Random solution construction



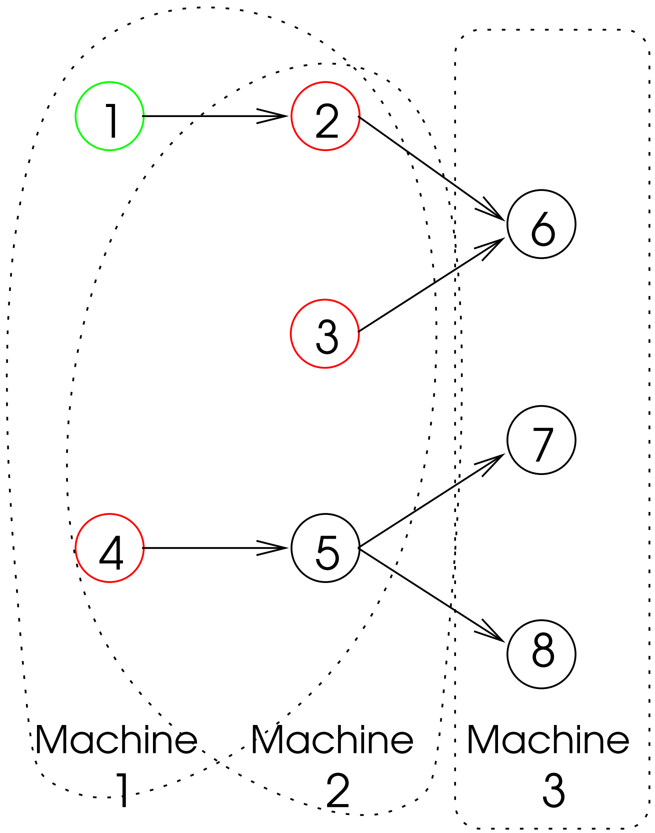
- Check all operations that can be scheduled

Random solution construction



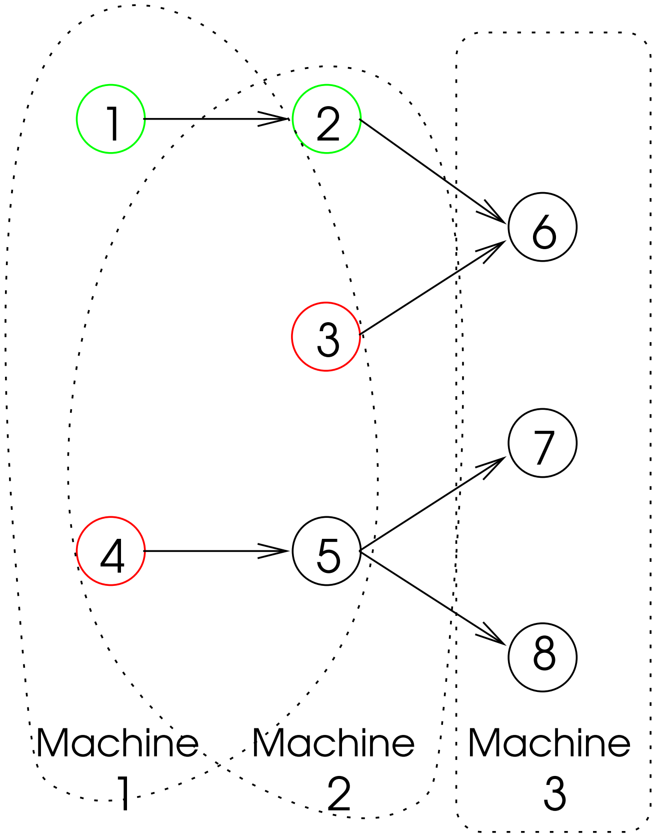
- Check all operations that can be scheduled
- Randomly select one of them (operation 1)
- Randomly select one of the compatible machines (machine 1)
- Fix this operation

Random solution construction



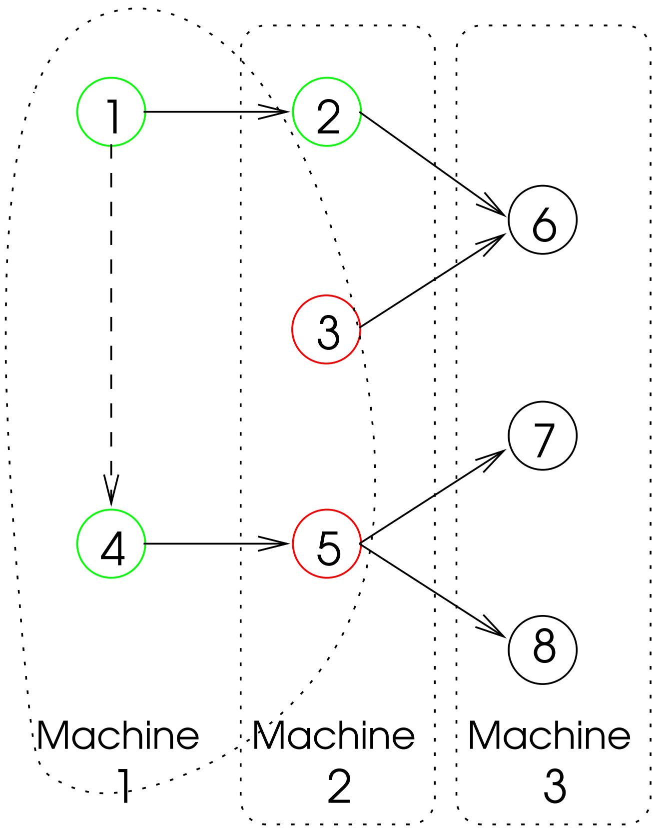
- (operation 1 is fixed on machine 1)
- Check all operations that can be scheduled (operations 2, 3, 4)
- Randomly select one of them (operation 2)
- Randomly select one of the compatible machines (machine 2)
- Fix this operation

Random solution construction



- (operation 1 is fixed on machine 1)
- (operation 2 is fixed on machine 2)
- Check all operations that can be scheduled (opertions 3, 4)
- Randomly select one of them (operation 4)
- Randomly select one of the compatible machines (machine 1)
- Fix this operation

Random solution construction

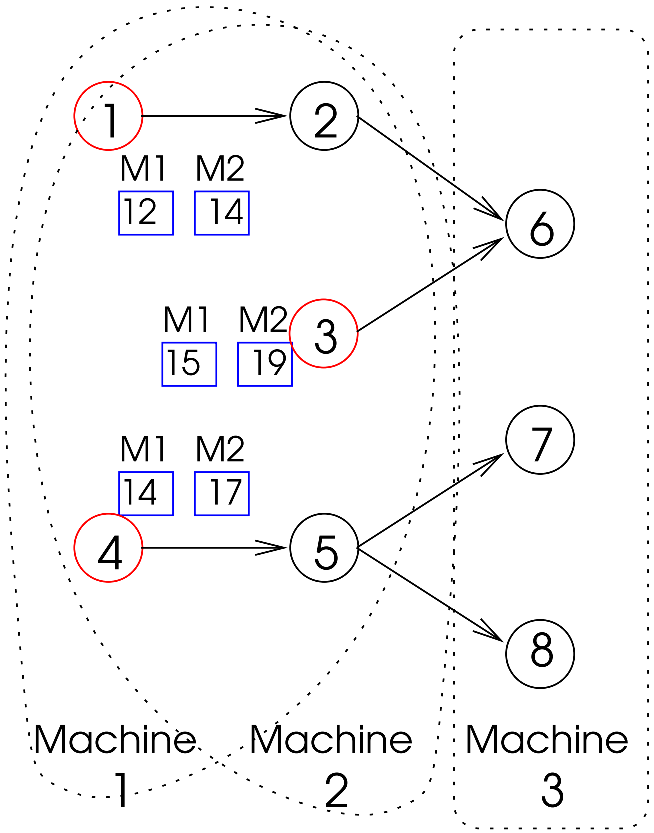


- (operation 1 is fixed on machine 1)
- (operation 2 is fixed on machine 2)
- (operation 4 is fixed on machine 1)
- Check all operations that can be scheduled (opertions 3, 5)
- Randomly select one of them . . .
- Randomly select one of the compatible machines . . .
- . . .
- Until all operations are scheduled

Random solution construction

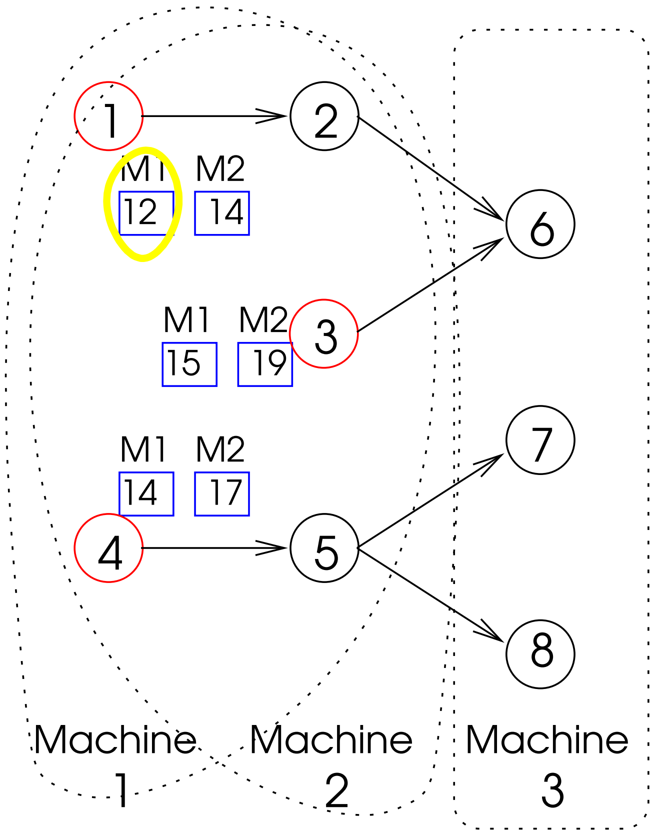
- Produces a random, but feasible solution (except for violation of maximal makespan)
- Very easy to implement
- Can produce many different solutions
- If repeated many times: might obtain a good solution

Greedy construction



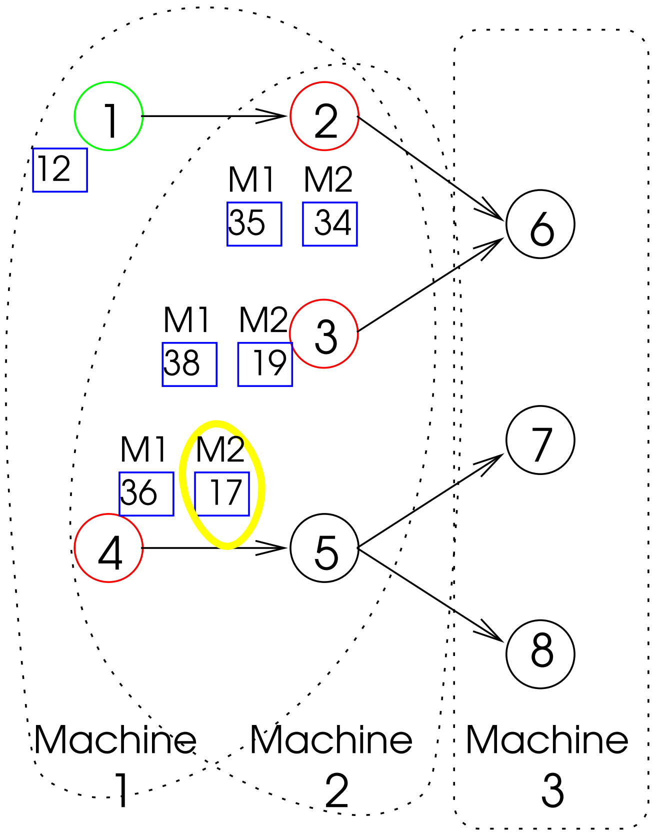
- Check all operations that can be scheduled
- Compute the *current makespan* when they are assigned to each of the possible machines

Greedy construction



- Check all operations that can be scheduled
- Compute the *current makespan* when they are assigned to each of the possible machines
- Select the assignment which induces the *smallest* makespan
- Fix this operation

Greedy construction



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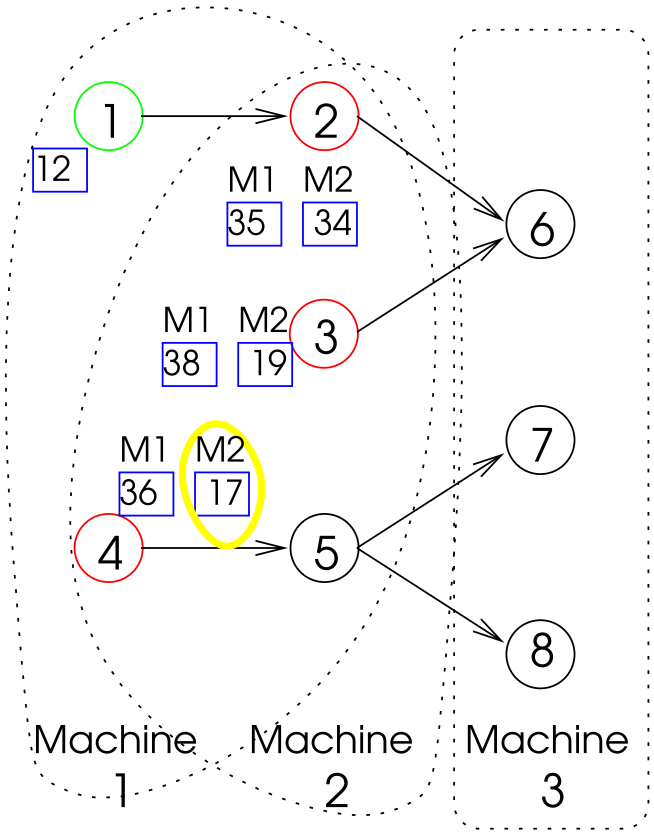
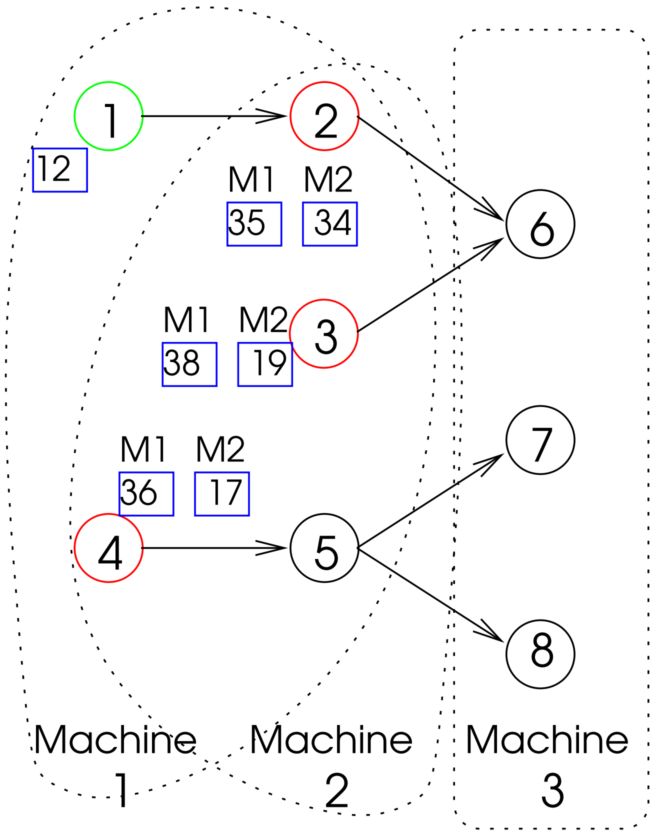
Greedy construction



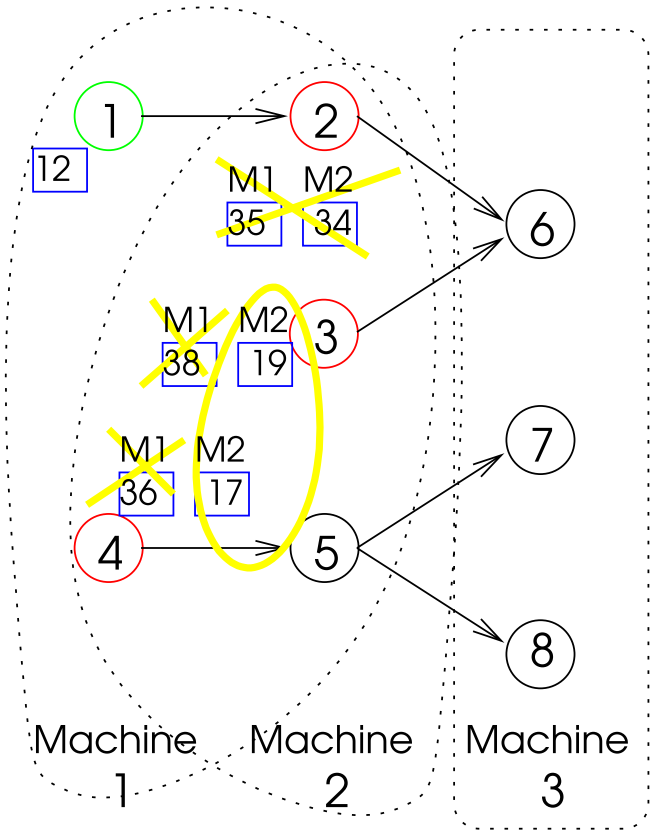
- Check all operations that can be scheduled
- Compute the *current makespan* when they are assigned to each of the possible machines
- Select the assignment which induces the *smallest* makespan
- Fix this operation
- . . .
- Continue this way until fixing all the operations

Semi-greedy construction



- As in the greedy construction, we check *all the possibilities* for each operation the can be scheduled
- Compute the current makespan for each of these possibilities

Semi-greedy construction

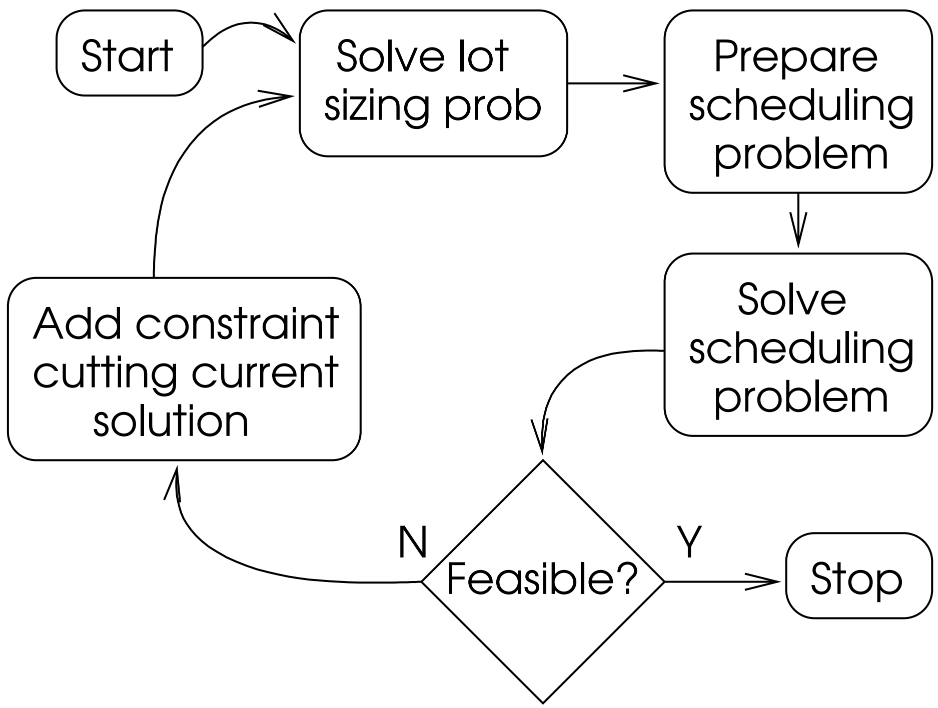


- As in the greedy construction, we check *all the possibilities* for each operation the can be scheduled
- Compute the current makespan for each of these possibilities
- Then, select just the possibilities that satisfy some criterion
- Create a *Restricted Candidate List* (RCL)
- Randomly select an (operation, machine) pair from the RCL
- Fix that operation on that machine
- . . .
- Continue, until fixing all the operations

An algorithm for repeated construction

```
IteratedSemiGreedy(N, t̄)
t* = ∞
c* = ∞
for n = 1 to N
    x = SemiGreedyConstruct()
    t = Makespan(x)
    c = Cost(x)
    if (t < t̄ and c < c*) or (t < t* and t* > t̄)
        x* = x; t* = t; c* = c
return x*
```

Main solution procedure (integration)



“No good” cuts

Let

- $y \in \{0, 1\}$ be a partial MIP solution
- $S = \{r : y_r = 1\}$ represent an assignment of tasks to machines

Then, if scheduling cannot find a feasible solution, add cut:

$$\sum_{r \in S} y_r \leq |S| - 1$$

Cuts for capacity adjustment

Let

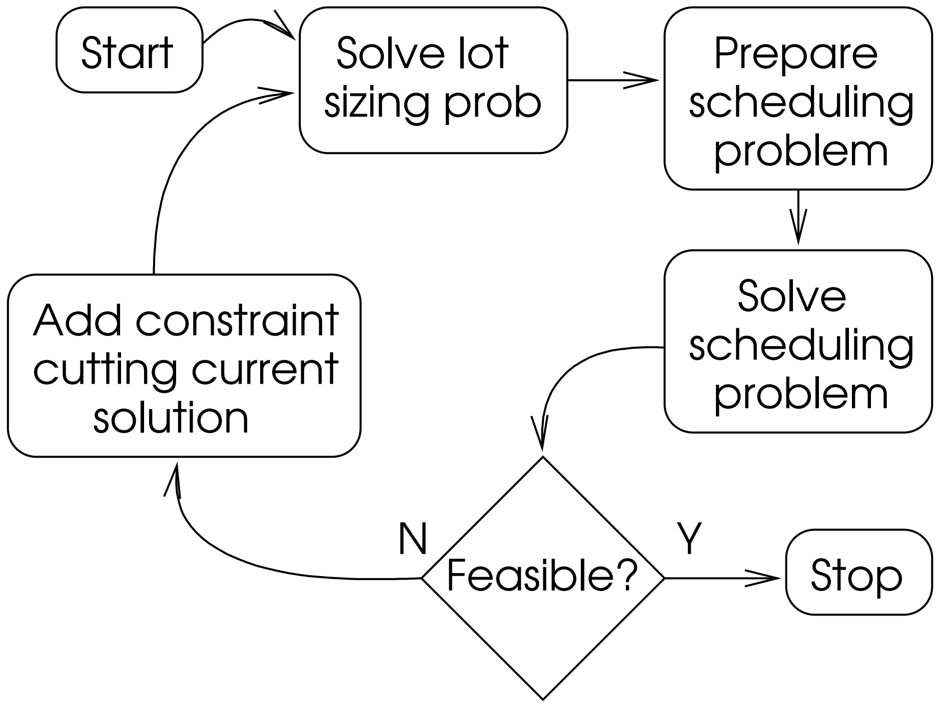
- x_{pmt} be the production of item p on period t, machine k
- $\bar{x}_{pmt}$ last MIP solution for these variables
- I_{mt} the heuristic estimate of machine waiting times

If we cannot find a feasible schedule of the tasks on period t, then

- for the set of machines M^* which did not respect the allowed makespan
- add cut:

$$\sum_{p \in P_m} x_{pmt} \leq \sum_{p \in P_m} \bar{x}_{pmt} - I_{mt} \qquad \forall m \in M^*$$

Main solution procedure



Conclusion

- Motivation: industrial application on production planning
- Lot sizing and scheduling: exact solution difficult for both problems
- Integrated model: even more difficult
- Integration of the models has in itself a heuristic component
- Proposed metaheuristics: there is potential for improvement, but
- The method quickly provides implementable solutions
- Results are sufficient for the current practical requirements